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 **$CP$  Violation in  $B_d \rightarrow D^+ D^-$ ,  $D^{*+} D^-$ ,  $D^+ D^{*-}$  and  $D^{*+} D^{*-}$  Decays****Zhi-zhong Xing**<sup>1</sup>*Sektion Physik, Universität München, Theresienstrasse 37A, 80333 München, Germany***Abstract**

$CP$  asymmetries in  $B_d \rightarrow D^+ D^-$ ,  $D^{*+} D^-$ ,  $D^+ D^{*-}$  and  $D^{*+} D^{*-}$  decays are investigated with the help of the factorization approximation and isospin relations. We find that the direct  $CP$  violation is governed only by the short-distance penguin mechanism, while the indirect  $CP$  asymmetries in  $B_d \rightarrow D^\pm D^{*\mp}$  transitions may be modified due to the final-state rescattering effect. An updated numerical analysis shows that the direct  $CP$  asymmetry in  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow D^+ D^-$  decays can be as large as 3%. The  $CP$ -even and  $CP$ -odd contributions to the indirect  $CP$  asymmetry in  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow D^{*+} D^{*-}$  decays are found to have the rates 89% and 11%, respectively. Some comments on the possibilities to determine the weak phase  $\beta$  and to test the factorization hypothesis are also given.

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# 1 Introduction

A direct measurement of the  $CP$ -violating parameter  $\sin 2\beta$  in  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow J/\psi K_S$  decays, where  $\beta \equiv \arg[-(V_{tb}^* V_{td})/(V_{cb}^* V_{cd})]$  is known as an inner angle of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0, \quad (1)$$

has recently been reported by the CDF Collaboration [1]. The preliminary result  $\sin 2\beta = 0.79_{-0.44}^{+0.41}$  (stat + syst) is consistent very well with the standard-model prediction for  $\sin 2\beta$ , obtained indirectly from a global analysis of current data on  $|V_{ub}/V_{cb}|$ ,  $B_d^0$ - $\bar{B}_d^0$  mixing, and  $CP$  violation in  $K^0$ - $\bar{K}^0$  mixing [3]. If the CDF measurement is confirmed,  $CP$  violation of the magnitude  $\sin 2\beta$  should also be seen in the decay modes  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow D^+ D^-$ ,  $D^{*+} D^-$ ,  $D^+ D^{*-}$  and  $D^{*+} D^{*-}$ , whose branching ratios are all anticipated to be of  $O(10^{-4})$ . Indeed the channel  $B_d^0 \rightarrow D^{*+} D^{*-}$  has been observed by the CLEO Collaboration [2], and the measured branching ratio  $\mathcal{B}(D^{*+} D^{*-}) = [6.2_{-2.9}^{+4.0}$  (stat)  $\pm 1.0$  (syst)]  $\times 10^{-4}$  is in agreement with the standard-model expectation. Further measurements of neutral and charged  $B$  decays into  $D^{(*)} \bar{D}^{(*)}$  states will soon be available in the first-round experiments of KEK and SLAC  $B$ -meson factories as well as at other high-luminosity hadron machines (see, e.g., Ref. [4] for a review with extensive references).

In the literature some special attention has been paid to  $B \rightarrow D^{(*)} \bar{D}^{(*)}$  transitions and  $CP$  violation. For example, the  $CP$  properties of  $B_d \rightarrow D^{(*)+} D^{(*)-}$  decays were analyzed in the heavy quark limit in Ref. [5]; the isospin relations and penguin effects in  $B \rightarrow D^{(*)} \bar{D}^{(*)}$  decays were explored in Ref. [6]; the possibility of extracting the weak phase  $\beta$  and testing the factorization hypothesis in  $B_d^0$  vs  $\bar{B}_d^0$  decays into the non- $CP$  eigenstates  $D^\pm D^{*\mp}$  were investigated in Ref. [7]; and the angular analysis of  $B_d \rightarrow D^{*+} D^{*-}$  decays to determine  $CP$ -even and  $CP$ -odd amplitudes were presented in Ref. [8]. In addition to those works, numerical estimates of branching ratios and  $CP$  asymmetries in  $B \rightarrow D^{(*)} \bar{D}^{(*)}$  decays have been given in Ref. [9], in which neither electroweak penguin contributions nor final-state rescattering effects were taken into account.

The present paper, different in several aspects from those previous studies, aims at analyzing final-state rescattering effects on direct and indirect  $CP$  asymmetries in  $B \rightarrow D^{(*)} \bar{D}^{(*)}$  decays. We calculate the  $I = 1$  and  $I = 0$  isospin amplitudes of these processes by using the factorization approximation and the effective weak Hamiltonian, and account for long-distance interactions at the hadron level by introducing elastic rescattering phases for two isospin channels of the final-state mesons. In this approach we find that direct  $CP$  asymmetries in both charged and neutral  $B$  decay modes are governed only by the short-distance penguin mechanism, but indirect  $CP$  asymmetries in  $B_d \rightarrow D^\pm D^{*\mp}$  transitions may be modified due to the final-state rescattering effect. An updated numerical analysis of direct  $CP$  violation in  $B \rightarrow D \bar{D}$ ,  $D^* \bar{D}$ ,  $D \bar{D}^*$  and  $D^* \bar{D}^*$  decays is made without neglect of the

electroweak penguin effects. We obtain the asymmetry as large as 3% in  $B_u^+ \rightarrow D^+ \bar{D}^0$  vs  $B_u^- \rightarrow D^- D^0$  or  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow D^+ D^-$  decays. In the absence of angular analysis we find that the indirect  $CP$  asymmetry in  $B_d \rightarrow D^{*+} D^{*-}$  decays is diluted by a factor 0.89, i.e., 11% of the asymmetry arising from the  $P$ -wave ( $CP$ -odd) contribution. We also give some comments on the possibilities to determine the weak phase  $\beta$  and to test the factorization hypothesis in the presence of final-state interactions.

## 2 Isospin amplitudes

The effective weak Hamiltonian responsible for  $B \rightarrow D^{(*)} \bar{D}^{(*)}$  decays can explicitly be written as [10]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_q \left[ V_{qb} V_{qd}^* \left( \sum_{i=1}^2 c_i Q_i^q + \sum_{i=3}^{10} c_i Q_i \right) \right] + \text{h.c.} , \quad (2)$$

where  $V_{qb}$  and  $V_{qd}$  (for  $q = u, c$ ) are the CKM matrix elements,  $c_i$  (for  $i = 1, \dots, 10$ ) are the Wilson coefficients, and

$$\begin{aligned} Q_1^q &= (\bar{d}_\alpha q_\beta)_{V-A} (\bar{q}_\beta b_\alpha)_{V-A} , \\ Q_2^q &= (\bar{d} q)_{V-A} (\bar{q} b)_{V-A} , \\ Q_3 &= (\bar{d} b)_{V-A} (\bar{c} c)_{V-A} , \\ Q_4 &= (\bar{d}_\alpha b_\beta)_{V-A} (\bar{c}_\beta c_\alpha)_{V-A} , \\ Q_5 &= (\bar{d} b)_{V-A} (\bar{c} c)_{V+A} , \\ Q_6 &= (\bar{d}_\alpha b_\beta)_{V-A} (\bar{c}_\beta c_\alpha)_{V+A} , \end{aligned} \quad (3)$$

as well as  $Q_7 = Q_5$ ,  $Q_8 = Q_6$ ,  $Q_9 = Q_3$  and  $Q_{10} = Q_4$ . Here  $Q_3, \dots, Q_6$  denote the QCD-induced penguin operators, and  $Q_7, \dots, Q_{10}$  stand for the electroweak penguin operators.

It is clear that the  $\Delta B = +1$  and  $\Delta B = -1$  parts of  $\mathcal{H}_{\text{eff}}$  have the isospin structures  $|1/2, +1/2\rangle$  and  $|1/2, -1/2\rangle$ , respectively. They govern the transitions  $B_u^+ \rightarrow D^{(*)+} \bar{D}^{(*)0}$ ,  $B_d^0 \rightarrow D^{(*)+} D^{*-}$ ,  $B_d^0 \rightarrow D^{(*)0} \bar{D}^{(*)0}$  and their  $CP$ -conjugate processes. The final state of each decay mode can be in either  $I = 1$  or  $I = 0$  isospin configuration. For simplicity we denote the amplitudes of six relevant transitions by use of the electric charges of their final- and initial-state mesons, i.e.,  $A^{+0}$ ,  $A^{+-}$ ,  $A^{00}$  (for  $B_u^+$  and  $B_d^0$  decays) and  $\bar{A}^{-0}$ ,  $\bar{A}^{+-}$ ,  $\bar{A}^{00}$  (for  $B_u^-$  and  $\bar{B}_d^0$  decays). These amplitudes can be expressed in terms of the  $I = 1$  and  $I = 0$  isospin amplitudes, which include both weak and strong phases. For example [6]<sup>2</sup>,

$$\begin{aligned} A^{+0} &= A_1 , \\ A^{+-} &= \frac{1}{2}(A_1 + A_0) , \\ A^{00} &= \frac{1}{2}(A_1 - A_0) ; \end{aligned} \quad (4)$$

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<sup>2</sup>As for  $B \rightarrow D^* \bar{D}^*$  decays, the isospin relations hold separately for the transition amplitudes with helicity  $\lambda = -1, 0$  or  $+1$ .

and the relations between  $(\bar{A}^{-0}, \bar{A}^{+-}, \bar{A}^{00})$  and  $(\bar{A}_1, \bar{A}_0)$  hold in the same form. In the complex plane two sets of isospin relations form two triangles:  $A^{+0} = A^{+-} + A^{00}$  and  $\bar{A}^{-0} = \bar{A}^{+-} + \bar{A}^{00}$ .

To calculate the magnitudes of  $I = 1$  and  $I = 0$  isospin amplitudes, we make use of the effective Hamiltonian  $\mathcal{H}_{\text{eff}}$  and the factorization approximation. We neglect the contributions of the annihilation-type channels, which are expected to have significant form-factor suppression [11]. It should be noted that in this approach the Wilson coefficients and the relevant hadronic matrix elements of four-quark operators need be evaluated in the same renormalization scheme and at the same energy scale. Following the procedure described in Ref. [12] one can obtain the scale- and renormalization-scheme-independent transition amplitudes consisting of the CKM factors, the effective Wilson coefficients, the penguin loop-integral functions and the factorized hadronic matrix elements. Under isospin symmetry, we are only left with two different hadronic matrix elements:

$$\begin{aligned} Z &= \langle D^{(*)+} | (\bar{c}d)_{V-A} | 0 \rangle \langle D^{(*)-} | (\bar{b}c)_{V-A} | B_d^0 \rangle \\ &= \langle D^{(*)+} | (\bar{c}d)_{V-A} | 0 \rangle \langle \bar{D}^{(*)0} | (\bar{b}c)_{V-A} | B_u^+ \rangle, \\ \bar{Z} &= \langle D^{(*)-} | (\bar{d}c)_{V-A} | 0 \rangle \langle D^{(*)+} | (\bar{c}b)_{V-A} | \bar{B}_d^0 \rangle \\ &= \langle D^{(*)-} | (\bar{d}c)_{V-A} | 0 \rangle \langle D^{(*)0} | (\bar{c}b)_{V-A} | B_u^- \rangle. \end{aligned} \quad (5)$$

Note that  $|\bar{Z}| = |Z|$  holds for the final states with two pseudoscalar mesons or those with one pseudoscalar and one vector mesons. Only for the final states with two vector mesons  $|\bar{Z}|$  and  $|Z|$  are different, as the  $P$ -wave contributions to  $Z$  and  $\bar{Z}$  have the opposite signs (see section 4 for the detail). Furthermore, we account for final-state interactions at the hadron level by introducing the elastic rescattering phases  $\delta_1$  and  $\delta_0$  for  $I = 1$  and  $I = 0$  isospin channels (a similar treatment can be found, e.g., in Refs. [13, 14]). We then arrive at the factorized isospin amplitudes as follows:

$$\begin{aligned} A_1 &= \frac{G_F}{\sqrt{2}} (V_{ud}V_{ub}^*S_u + V_{cd}V_{cb}^*S_c) Z e^{i\delta_1}, \\ A_0 &= \frac{G_F}{\sqrt{2}} (V_{ud}V_{ub}^*S_u + V_{cd}V_{cb}^*S_c) Z e^{i\delta_0}, \end{aligned} \quad (6)$$

in which  $S_u$  and  $S_c$  are composed of the effective Wilson coefficients and the penguin loop-integral functions (see section 3). The expressions of  $\bar{A}_1$  and  $\bar{A}_0$  can be obtained respectively from those of  $A_1$  and  $A_0$  in Eq. (6) through the replacements  $Z \Rightarrow \bar{Z}$  and  $V_{qd}V_{qb}^* \Rightarrow V_{qd}^*V_{qb}$  (for  $q = u$  and  $c$ ). Note that all parameters in the isospin amplitudes, except the CKM factors, are dependent upon the specific final states of  $B$  decays.

One can see that  $|A_0| = |A_1|$  and  $|\bar{A}_0| = |\bar{A}_1|$  hold in the context of our simple factorization scheme. This implies that the  $B_d \rightarrow D^{(*)0} \bar{D}^{(*)0}$  transitions would be forbidden, if there were no final-state rescattering effects (i.e., if  $\delta_0 = \delta_1$ ). Substituting Eq. (6) into Eq. (4),

one obtains

$$\begin{aligned} A^{+-} &= \frac{G_F}{\sqrt{2}} (V_{ud}V_{ub}^*S_u + V_{cd}V_{cb}^*S_c) Z \cos \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2}, \\ A^{00} &= i \frac{G_F}{\sqrt{2}} (V_{ud}V_{ub}^*S_u + V_{cd}V_{cb}^*S_c) Z \sin \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2}. \end{aligned} \quad (7)$$

Similarly  $\bar{A}^{+-}$  and  $\bar{A}^{00}$  can be read off from  $A^{+-}$  and  $A^{00}$  through the replacements  $Z \Rightarrow \bar{Z}$  and  $V_{qd}V_{qb}^* \Rightarrow V_{qd}^*V_{qb}$  (for  $q = u$  and  $c$ ). It is easy to find

$$\begin{aligned} |A^{+-}|^2 + |A^{00}|^2 &= |A^{+0}|^2, \\ |\bar{A}^{+-}|^2 + |\bar{A}^{00}|^2 &= |\bar{A}^{-0}|^2; \end{aligned} \quad (8)$$

i.e., the two isospin triangles are right-angled triangles. Whether the relations in Eq. (8) are practically valid or not can be checked, once the experimental data on branching ratios of  $B \rightarrow D^{(*)}\bar{D}^{(*)}$  decays are available.

If  $|A^{+-}| = |A^{+0}|$  held,  $|A^{00}| = 0$  would result within the factorization approach described above. Namely, observation of the (approximate) equality between the decay rates of  $B_d^0 \rightarrow D^{(*)+}D^{(*)-}$  and  $B_u^+ \rightarrow D^{(*)+}\bar{D}^{(*)0}$  would imply that the decay modes  $B_d^0 \rightarrow D^{(*)0}\bar{D}^{(*)0}$  were forbidden or strongly suppressed. This conclusion is in general not true, however. Without any special assumption or approximation, we denote  $A_0/A_1 = ze^{i\theta}$ ,  $|A^{00}/A^{+0}|^2 = R$  and obtain consequences of the equality  $|A^{+-}| = |A^{+0}|$  as follows:

$$\begin{aligned} z &= \sqrt{3 + \cos^2 \theta} - \cos \theta, \\ R &= 1 + \cos^2 \theta - \cos \theta \sqrt{3 + \cos^2 \theta}. \end{aligned} \quad (9)$$

The behaviors of  $z$  and  $R$  changing with  $\theta$  is illustrated in Fig. 1. It is clear that in general  $|A^{00}| = 0$  (i.e.,  $R = 0$ ) is not necessary to hold even if  $|A^{+-}| = |A^{+0}|$  holds. Therefore the detection of  $B_d \rightarrow D^{(*)0}\bar{D}^{(*)0}$  transitions is very useful in experiments, in order to demonstrate whether final-state rescattering effects are significant and to test whether the factorization approximation works well.

### 3 Direct $CP$ asymmetries

We proceed with the factorization scheme to calculate direct  $CP$  asymmetries in the decay modes under discussion. As for the final states with two vector mesons, we sum over their polarizations and arrive at  $|\bar{Z}|^2 = |Z|^2$ , a relationship which apparently holds for other types of final states. With the help of Eqs. (6) and (7) it is easy to show that the decay rate asymmetry between  $B_u^+ \rightarrow D^{(*)+}\bar{D}^{(*)0}$  and  $B_u^- \rightarrow D^{(*)-}D^{(*)0}$  decays is identical to that between  $B_d^0$  and  $\bar{B}_d^0 \rightarrow D^{(*)+}D^{(*)-}$  or  $D^{(*)0}\bar{D}^{(*)0}$  decays. All these  $CP$  asymmetries are

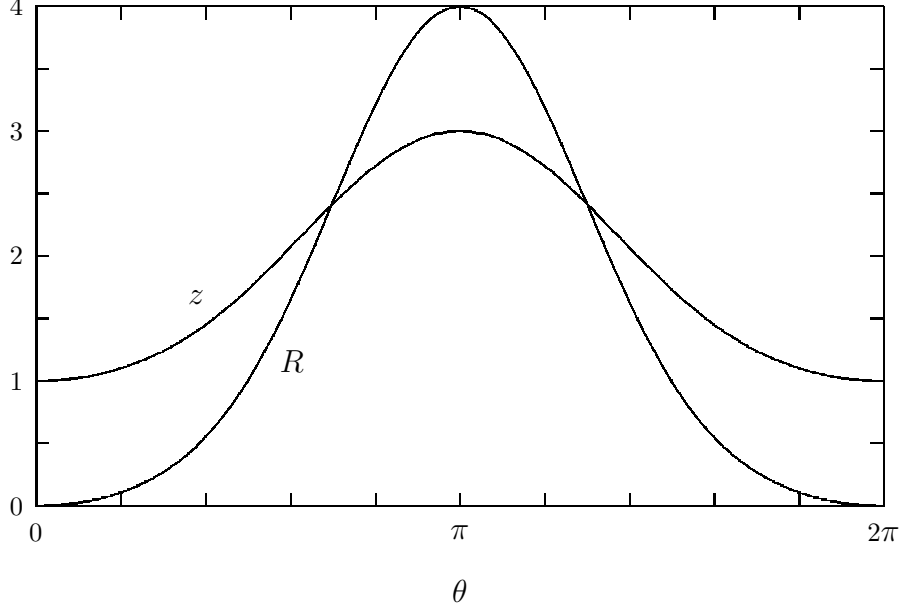


Figure 1: Behaviors of  $z$  and  $R$  changing with  $\theta$  under the condition  $|A^{+-}| = |A^{+0}|$ .

independent of the rescattering phases and the hadronic matrix elements <sup>3</sup>:

$$\begin{aligned}
\mathcal{A} &= \frac{|\bar{A}^{-0}|^2 - |A^{+0}|^2}{|\bar{A}^{-0}|^2 + |A^{+0}|^2} \\
&= \frac{|\bar{A}^{+-}|^2 - |A^{+-}|^2}{|\bar{A}^{+-}|^2 + |A^{+-}|^2} \\
&= \frac{|\bar{A}^{00}|^2 - |A^{00}|^2}{|\bar{A}^{00}|^2 + |A^{00}|^2} \\
&= \frac{2r \sin \gamma \text{Im}(\zeta_u \zeta_c^*)}{r^2 |\zeta_u|^2 + |\zeta_c|^2 - 2r \cos \gamma \text{Re}(\zeta_u \zeta_c^*)} , \tag{10}
\end{aligned}$$

where  $r$  and  $\gamma$  are defined by  $re^{i\gamma} \equiv -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ . The phase  $\gamma$  corresponds to another inner angle of the unitarity triangle defined in Eq. (1). Eq. (10) indicates that direct  $CP$  violation arises only from final-state interactions of the quark level (through the penguin mechanism) in  $B \rightarrow D^{(*)}\bar{D}^{(*)}$  decays. This result, as a straightforward consequence of the factorization approximation, can directly be confronted with the upcoming experiments at  $B$ -meson factories.

Let us evaluate the direct  $CP$  asymmetries  $\mathcal{A}$  for different final states. As mentioned above,  $S_u$  and  $S_c$  in Eq. (10) depend on the effective Wilson coefficients  $\bar{c}_i$  and the penguin loop-integral functions  $F_q$ . The latter can be given, for a momentum-squared transfer  $k^2$  at

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<sup>3</sup>For time-integrated  $B_d$  decays the direct  $CP$  asymmetries are diluted by a well-known factor  $1/(1+x_d^2)$ , where  $x_d \approx 0.7$  is the  $B_d^0\text{-}\bar{B}_d^0$  mixing parameter. In this paper we do not take such mixing effects into account.

the  $O(m_b)$  scale, as follows [15]:

$$F_q = 4 \int_0^1 dx \, x(1-x) \ln \left[ \frac{m_q^2 - k^2 x(1-x)}{m_b^2} \right]. \quad (11)$$

The absorptive part of  $F_q$ , which is a necessary condition for direct  $CP$  violation, emerges if  $k^2 \geq 4m_q^2$ . The concrete expressions of  $S_u$  and  $S_c$  are found to be

$$\begin{aligned} S_u &= C_1 + C_3 + C_4 \frac{1+\xi}{9\pi} \left( \frac{10}{3} + F_u \right), \\ S_c &= C_2 + C_3 + C_4 \frac{1+\xi}{9\pi} \left( \frac{10}{3} + F_c \right), \end{aligned} \quad (12)$$

where

$$\begin{aligned} C_1 &= \frac{\bar{c}_3}{3} + \bar{c}_4 + \frac{\bar{c}_9}{3} + \bar{c}_{10}, \\ C_2 &= \frac{\bar{c}_1}{3} + \bar{c}_2 + C_1, \\ C_3 &= \frac{\bar{c}_5}{3} + \bar{c}_6 + \frac{\bar{c}_7}{3} + \bar{c}_8, \\ C_4 &= \bar{c}_2 \alpha_s + \left( \bar{c}_1 + \frac{\bar{c}_2}{3} \right) \alpha_e. \end{aligned} \quad (13)$$

In these equations  $\bar{c}_i$  (for  $i = 1, \dots, 10$ ) are the renormalization-scheme-independent Wilson coefficients,  $\alpha_s$  and  $\alpha_e$  stand respectively for the strong and electroweak coupling constants, and  $\xi$  is a factorization parameter arising from the transformation of (V-A)(V+A) currents into (V-A)(V-A) ones for the penguin operators  $Q_5, \dots, Q_8$ . Note that  $\xi$  depends on properties of the final-state mesons [16]:

$$\xi = \begin{cases} +\frac{2m_D^2}{(m_c + m_d)(m_b - m_c)} & (D\bar{D}) \\ 0 & (D^*\bar{D}) \\ -\frac{2m_D^2}{(m_c + m_d)(m_b + m_c)} & (D\bar{D}^*) \\ 0 & (D^*\bar{D}^*) \end{cases}, \quad (14)$$

where the order of two  $D^{(*)}$  mesons corresponds to that in the factorized hadronic matrix element  $Z$  or  $\bar{Z}$ , as given in Eq. (5).

With the help of Eqs. (11) – (14) we are able to calculate the  $CP$  asymmetries  $\mathcal{A}$  numerically. Note that  $|S_u| \ll |S_c|$ , as the former consists only of the penguin contribution and the latter is dominated by the much larger tree-level contribution. This, together with  $|r| < 1$ , allows an instructive analytical approximation of  $\mathcal{A}$ :

$$\mathcal{A} \approx 2r \sin \gamma \text{Im} \left( \frac{S_u}{S_c} \right). \quad (15)$$

For illustration, we typically choose  $m_u = 5$  MeV,  $m_c = 1.35$  GeV,  $m_b = 5$  GeV and  $m_t = 174$  GeV. The strong coupling constant is taken as  $\alpha_s = 0.21$  at the  $O(m_b)$  scale. Values of

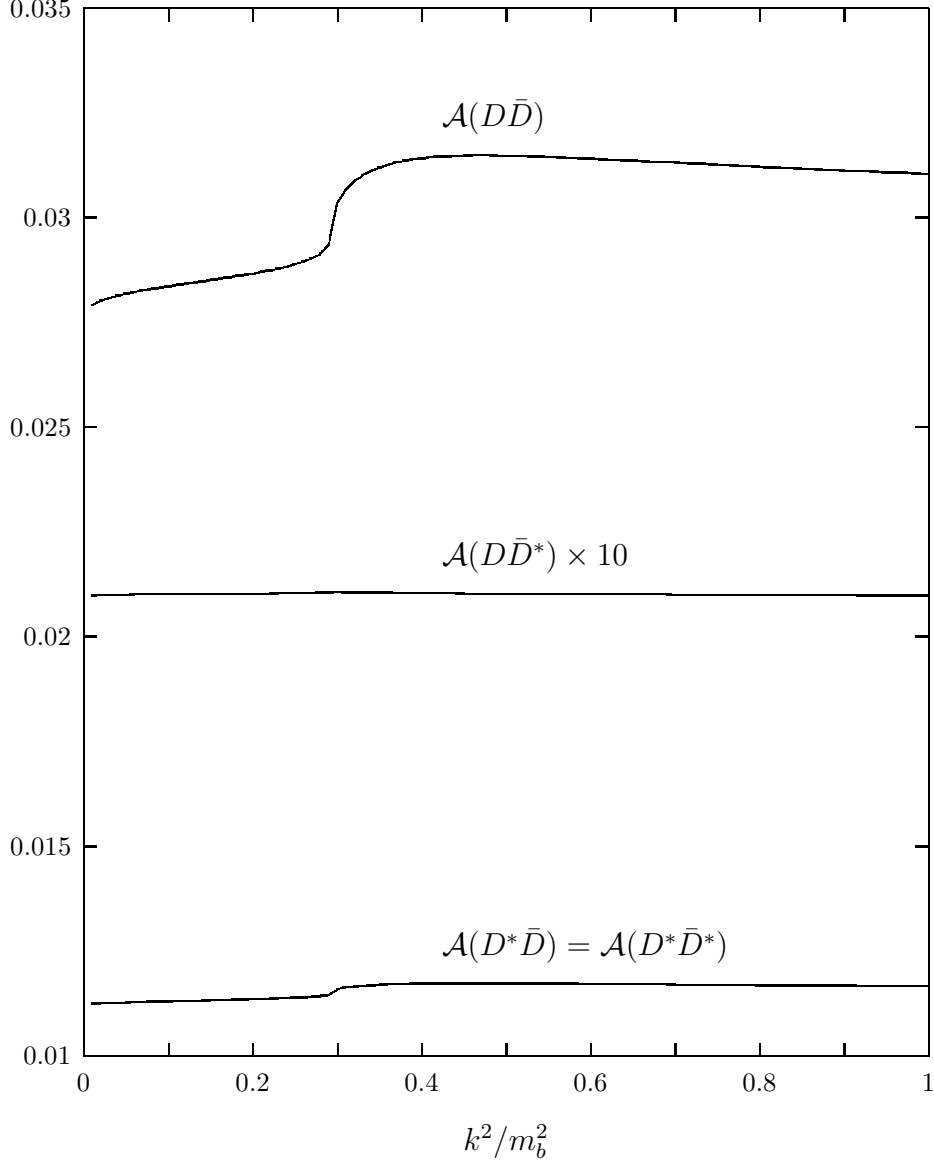


Figure 2: Direct  $CP$  asymmetries of  $B \rightarrow D^{(*)}\bar{D}^{(*)}$  in the factorization approximation.

the effective coefficients  $\bar{c}_i$  read [17]:  $\bar{c}_1 = -0.313$ ,  $\bar{c}_2 = 1.150$ ,  $\bar{c}_3 = 0.017$ ,  $\bar{c}_4 = -0.037$ ,  $\bar{c}_5 = 0.010$ ,  $\bar{c}_6 = -0.046$ ,  $\bar{c}_7 = -0.001\alpha_e$ ,  $\bar{c}_8 = 0.049\alpha_e$ ,  $\bar{c}_9 = -1.321\alpha_e$  and  $\bar{c}_{10} = 0.267\alpha_e$  with  $\alpha_e = 1/128$ . The CKM factors are taken to be  $r = 0.38$  and  $\gamma = 60^\circ$ , consistent with the latest data on quark mixing and  $CP$  violation [3]. The unknown penguin momentum transfer  $k^2$  is treated as a free parameter changing from  $0.01m_b^2$  to  $m_b^2$ . Our numerical results are shown in Fig. 2. Some discussions are in order.

1. All  $CP$  asymmetries have the same sign and undergo a change of magnitude at  $k^2 = 4m_c^2 \approx 0.3m_b^2$ . The asymmetry  $\mathcal{A}(D\bar{D})$  is most sensitive to the uncertain penguin momentum transfer  $k^2$ , but its magnitude increases only about 0.3% from  $k^2 = 0.01m_b^2$  to  $k^2 = m_b^2$ . It is found that the strong (gluonic) penguin effect is dominant over the electroweak penguin effect, thus the latter is safely negligible.



2. The  $CP$  asymmetry  $\mathcal{A}(D\bar{D})$  can be as large as 3%, while  $\mathcal{A}(D\bar{D}^*)$  is only about  $2 \times 10^{-3}$ . The smallness of the latter comes from the cancellation effect, induced by the factor  $(1 + \xi)$  with  $\xi \sim -0.8$ , in  $S_u$  and  $S_c$ . In our factorization approximation, the asymmetries  $\mathcal{A}(D^*\bar{D})$  and  $\mathcal{A}(D^*\bar{D}^*)$  are identical and of magnitude 1%.
3. Observation of the  $CP$  asymmetries  $\mathcal{A}(D\bar{D})$  and  $\mathcal{A}(D^*\bar{D}^*)$  to three standard deviations needs about  $10^8$   $B_u^\pm$  events, if the composite detection efficiency is at the 10% level. More events are required to measure the same  $CP$  asymmetries in  $B_d$  decays, due to the cost for flavor tagging.

It is therefore worth while to search for such direct  $CP$ -violating signals in the first-round experiments of  $B$ -meson factories.

## 4 Indirect $CP$ violation

Although the final-state rescattering phases have no effect on direct  $CP$  asymmetries  $\mathcal{A}$  in our factorization scheme, they are possible to influence the indirect  $CP$  violation arising from the interference between direct  $B_d$  transition and  $B_d^0$ - $\bar{B}_d^0$  mixing in the decay modes under consideration. The characteristic measurable of this source of  $CP$  violation is in general a difference between two rephasing-invariant quantities defined as [18]

$$\begin{aligned}\Delta(f) &= \text{Im} \left[ \frac{q}{p} \cdot \frac{A(\bar{B}_d^0 \rightarrow f)}{A(B_d^0 \rightarrow f)} \right], \\ \bar{\Delta}(\bar{f}) &= \text{Im} \left[ \frac{p}{q} \cdot \frac{A(B_d^0 \rightarrow \bar{f})}{A(\bar{B}_d^0 \rightarrow \bar{f})} \right],\end{aligned}\tag{16}$$

where  $q/p = (V_{tb}^* V_{td}) / (V_{tb} V_{td}^*)$  denotes the weak phase of  $B_d^0$ - $\bar{B}_d^0$  mixing <sup>4</sup>, and  $\bar{f}$  is the  $CP$ -conjugate state of  $f$ . If  $f$  is a  $CP$  eigenstate (i.e.,  $|\bar{f}\rangle = CP|f\rangle = \pm|f\rangle$ ) and the decay is dominated by the tree-level channel, then  $\bar{\Delta}(\bar{f}) = -\Delta(f)$  is a good approximation. In general only the difference  $\bar{\Delta}(\bar{f}) - \Delta(f)$ , which will vanish if all the CKM factors are real, measures the  $CP$  asymmetry. Note that the  $CP$ -even and  $CP$ -odd components of  $f = D^{*+}D^{*-}$  state or  $f = D^{*0}\bar{D}^{*0}$  state may cause some dilution in the measurables  $\Delta(f)$  and  $\bar{\Delta}(\bar{f})$ . A proper treatment of indirect  $CP$  violation in such modes is to make use of the angular analysis [8]. Alternatively one may evaluate the  $P$ -wave contribution to  $\Delta(D^{*+}D^{*-})$  and  $\Delta(D^{*0}\bar{D}^{*0})$  by use of the factorization approximation and the heavy quark symmetry [20].

As penguin contributions to the transition amplitudes of  $B \rightarrow D^{(*)}\bar{D}^{(*)}$  decays have been estimated to be at the percent level, we expect that their effects on  $\Delta(f)$  and  $\bar{\Delta}(\bar{f})$  are unimportant and negligible.

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<sup>4</sup>The  $CP$  violation induced solely by  $B_d^0$ - $\bar{B}_d^0$  mixing (i.e.,  $|q/p| \neq 1$ ) is expected to be negligibly small (of order  $10^{-3}$  or smaller [19]) in the standard model.

It is obvious, as shown in Eq. (7), that for  $B_d^0$  and  $\bar{B}_d^0$  decays into the  $CP$  eigenstates  $D^+D^-$  and  $D^0\bar{D}^0$  the amplitude ratios  $\bar{A}^{+-}/A^{+-}$  and  $\bar{A}^{00}/A^{00}$  are independent of the rescattering effects. Neglecting small penguin contributions to  $S_u$  and  $S_c$  (i.e., taking  $S_u = 0$  and  $S_c = \bar{c}_2 + \bar{c}_1/3$ ), we arrive at

$$\begin{aligned}\Delta(D^+D^-) &= \Delta(D^0\bar{D}^0) = +\sin 2\beta, \\ \bar{\Delta}(D^+D^-) &= \bar{\Delta}(D^0\bar{D}^0) = -\sin 2\beta,\end{aligned}\tag{17}$$

where  $\beta$  is just the inner angle of the unitarity triangle defined in Eq. (1). Therefore the measurement of indirect  $CP$  asymmetries in  $B_d \rightarrow D^+D^-$  and  $B_d \rightarrow D^0\bar{D}^0$  decays may serve as a cross-check of  $\sin 2\beta$  extracted from the  $CP$  asymmetry in  $B_d \rightarrow J/\psi K_S$  decays.

The channels  $B_d \rightarrow D^{*+}D^-$ ,  $D^+D^{*-}$  and  $B_d \rightarrow D^{*0}\bar{D}^0$ ,  $D^0\bar{D}^{*0}$ , whose final states are non- $CP$  eigenstates, are also useful for extraction of the weak angle  $\beta$ . Since the pseudoscalar and vector mesons from  $B_d^0$  and those from  $\bar{B}_d$  have different quark-diagram configurations, the hadronic matrix elements and final-state rescattering phases in these two processes should in general be different [7]. As a result,

$$\begin{aligned}\Delta(D^{*+}D^-) &= \zeta R_{+-} \sin(\delta + 2\beta), \\ \bar{\Delta}(D^+D^{*-}) &= \zeta R_{+-} \sin(\delta - 2\beta);\end{aligned}\tag{18}$$

and

$$\begin{aligned}\Delta(D^{*0}\bar{D}^0) &= \zeta R_{00} \sin(\delta + 2\beta), \\ \bar{\Delta}(D^0\bar{D}^{*0}) &= \zeta R_{00} \sin(\delta - 2\beta),\end{aligned}\tag{19}$$

where

$$\begin{aligned}\zeta &= \frac{Z_{D\bar{D}^*}}{Z_{D^*\bar{D}}} = \frac{f_D}{f_{D^*}} \cdot \frac{A_0^{BD^*}(m_D^2)}{F_1^{BD}(m_{D^*}^2)}, \\ \delta &= \frac{\delta_1^{D\bar{D}^*} + \delta_0^{D\bar{D}^*}}{2} - \frac{\delta_1^{D^*\bar{D}} + \delta_0^{D^*\bar{D}}}{2};\end{aligned}\tag{20}$$

and

$$\begin{aligned}R_{+-} &= \frac{\cos \frac{\delta_1^{D\bar{D}^*} - \delta_0^{D\bar{D}^*}}{2}}{\cos \frac{\delta_1^{D^*\bar{D}} - \delta_0^{D^*\bar{D}}}{2}}, \\ R_{00} &= \frac{\sin \frac{\delta_1^{D\bar{D}^*} - \delta_0^{D\bar{D}^*}}{2}}{\sin \frac{\delta_1^{D^*\bar{D}} - \delta_0^{D^*\bar{D}}}{2}}.\end{aligned}\tag{21}$$

In obtaining these results we have neglected the small penguin effects. The decay constants and form-factors in the expression of  $\zeta$ , coming from decomposition of the hadronic matrix

elements  $Z_{D\bar{D}^*}$  and  $Z_{D^*\bar{D}}$  given in Eq. (5), are self-explanatory. Note that  $R_{+-} = 1$  and  $\delta = \delta_1^{D\bar{D}^*} - \delta_1^{D^*\bar{D}}$  hold, if one takes the limit  $\delta_1^f = \delta_0^f$  (for each final state  $f$ ), in which the decay modes  $B_d \rightarrow D^{(*)0}\bar{D}^{(*)0}$  become forbidden. In the presence of rescattering effects, i.e.,  $R_{+-} \neq 1$ , the extraction of  $\beta$  from  $\Delta(D^{*+}D^-)$  and  $\bar{\Delta}(D^+D^{*-})$  seems difficult. However, it is possible to determine the isospin phase difference  $\delta_1^f - \delta_0^f$  from the triangle relation in Eq. (4), if the relevant rates of three (one charged  $B$  and two neutral  $B$ ) decay modes are measured in experiments. The observation of  $B_d \rightarrow D^{*0}\bar{D}^0$  and  $D^0\bar{D}^{*0}$  transitions turns out to be crucial: (a) if their branching ratios in comparison with those of  $B_d \rightarrow D^{*+}D^-$  and  $D^+D^{*-}$  decays are too small to be detected, then the final-state rescattering effects should be negligible and the naive factorization approach with  $R_{+-} = 1$  might work well; (b) if their branching ratios are more or less comparable with those of  $B_d \rightarrow D^{*+}D^-$  and  $D^+D^{*-}$  decays, then a quantitative isospin analysis should be available, allowing us to extract the isospin phase differences and determine the magnitudes of  $R_{+-}$  and  $R_{00}$ . In both cases,  $\zeta$  can experimentally be determined and the result can be confronted with the theoretical value of  $\zeta$  calculated by inputting relevant decay constants and form-factors.

For  $B_d \rightarrow D^{*+}D^{*-}$  and  $B_d \rightarrow D^{*0}\bar{D}^{*0}$  decay modes the indirect  $CP$  asymmetries need a more careful analysis. Note that the transition amplitude of  $B_d^0 \rightarrow D^{*+}D^{*-}$  (or  $D^{*0}\bar{D}^{*0}$ ) is a sum of three different components, i.e., the  $S$ -,  $D$ - and  $P$ -wave amplitudes [21]. Without loss of generality the hadronic matrix elements  $Z$  and  $\bar{Z}$  for  $B_d \rightarrow D^{*+}D^{*-}$  can be written as

$$\begin{aligned}
Z &= \tilde{a} (\epsilon_+ \cdot \epsilon_-) + \frac{\tilde{b}}{m_{D^*}^2} (p_0 \cdot \epsilon_+) (p_0 \cdot \epsilon_-) \\
&\quad + i \frac{\tilde{c}}{m_{D^*}^2} (\epsilon^{\alpha\beta\gamma\delta} \epsilon_{+\alpha} \epsilon_{-\beta} p_{+\gamma} p_{0\delta}) , \\
\bar{Z} &= \tilde{a} (\epsilon_+ \cdot \epsilon_-) + \frac{\tilde{b}}{m_{D^*}^2} (p_0 \cdot \epsilon_+) (p_0 \cdot \epsilon_-) \\
&\quad - i \frac{\tilde{c}}{m_{D^*}^2} (\epsilon^{\alpha\beta\gamma\delta} \epsilon_{+\alpha} \epsilon_{-\beta} p_{+\gamma} p_{0\delta}) ,
\end{aligned} \tag{22}$$

where  $\epsilon_{\pm}$  denotes the polarization of  $D^{*\pm}$  meson,  $p_0$  and  $p_{\pm}$  stand respectively for the momenta of  $B_d$  and  $D^{*\pm}$  mesons, and  $(\tilde{a}, \tilde{b}, \tilde{c})$  are real scalars without the penguin effects. In terms of the decay constants and form factors,  $\tilde{a}$ ,  $\tilde{b}$  and  $\tilde{c}$  read explicitly as

$$\begin{aligned}
\tilde{a} &= m_{D^*} f_{D^*} (m_B + m_{D^*}) A_1^{BD^*}(m_{D^*}^2) , \\
\tilde{b} &= -2m_{D^*}^3 f_{D^*} \frac{A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}} , \\
\tilde{c} &= -2m_{D^*}^3 f_{D^*} \frac{V^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}} .
\end{aligned} \tag{23}$$

In the absence of angular analysis one may first calculate the ratio  $\bar{Z}/Z$  by summing over the polarizations of two final-state vector mesons [21], and then calculate the  $CP$ -violating

quantities  $\Delta(D^{*+}D^{*-})$  and  $\bar{\Delta}(D^{*+}\bar{D}^{*-})$  in the neglect of small penguin effects. We finally arrive at

$$\begin{aligned}\Delta(D^{*+}D^{*-}) &= \Delta(D^{*0}\bar{D}^{*0}) = +\sin 2\beta \frac{1-\chi}{1+\chi}, \\ \bar{\Delta}(D^{*+}D^{*-}) &= \Delta(D^{*0}\bar{D}^{*0}) = -\sin 2\beta \frac{1-\chi}{1+\chi},\end{aligned}\tag{24}$$

where

$$\chi = \frac{2(x^2 - 1)\tilde{c}^2}{(2 + x^2)\tilde{a}^2 + (x^2 - 1)^2\tilde{b}^2 + 2x(x^2 - 1)\tilde{a}\tilde{b}}\tag{25}$$

with  $x = (m_B^2 - 2m_{D^*}^2)/(2m_{D^*}^2) = 2.45$ . Clearly the dilution parameter  $\chi$  results from the  $P$ -wave contribution to the overall decay amplitudes. If we adopt the simple monopole model for relevant form factors [22], it turns out that  $V^{BD^*}(m_{D^*}^2) = 0.784$ ,  $A_1^{BD^*}(m_{D^*}^2) = 0.715$  and  $A_2^{BD^*}(m_{D^*}^2) = 0.753$ . Accordingly  $\tilde{b}/\tilde{a} = -0.160$  and  $\tilde{c}/\tilde{a} = -0.167$ . The relationship  $\tilde{b}/\tilde{a} \approx \tilde{c}/\tilde{a}$  is indeed guaranteed by the heavy quark symmetry, which makes the form factors appearing in Eq. (23) related to one another. In this symmetry limit we obtain [20]

$$\frac{\tilde{b}}{\tilde{a}} = \frac{\tilde{c}}{\tilde{a}} = -\frac{2m_{D^*}^2}{m_B(m_B + 2m_{D^*})},\tag{26}$$

amounting to  $-0.164$ . Then we get  $(1 - \chi)/(1 + \chi) \approx 0.89$ , a value deviating only about 11% from unity. Note that this dilution factor can also be determined from measuring the ratio  $\Delta(D^{*+}D^{*-})/\Delta(D^+D^-)$ . From this estimation we find that the  $P$ -wave dilution effect is not very significant, therefore extracting the  $CP$ -violating parameter  $\sin 2\beta$  from  $B_d \rightarrow D^{*+}D^{*-}$  decays remains possible even if a delicate angular analysis is not made.

## 5 Summary

We have analyzed direct and indirect  $CP$  asymmetries in  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow D^+D^-$ ,  $D^{*+}D^-$ ,  $D^+D^{*-}$  and  $D^{*+}D^{*-}$  decays. The isospin amplitudes of these transitions are calculated with the help of the effective weak Hamiltonian and the factorization approximation, and the long-distance interactions at the hadron level are taken in to account by introducing elastic rescattering phases for two isospin channels of the final-state mesons. We have shown that in this factorization approach the direct  $CP$  violation is irrelevant to the final-state rescattering effects, i.e., it is governed only by the short-distance penguin mechanism. The magnitude of direct  $CP$  violation is estimated to be 3% in  $B_d \rightarrow D^+D^-$  decay modes. The same amount of  $CP$  violation can manifest itself in the charged  $B_u$  decays into  $D^+\bar{D}^0$  and  $D^-D^0$  states, which are easier to be measured at  $B$ -meson factories. We have demonstrated that the penguin effects on indirect  $CP$  asymmetries in  $B_d \rightarrow D^{(*)+}D^{(*)-}$  decays are insignificant and even negligible. While the long-distance rescattering has no effect on indirect  $CP$  violation in  $B_d \rightarrow D^+D^-$  and  $D^{*+}D^{*-}$  transitions, it may affect that in  $B_d \rightarrow D^\pm D^{*\mp}$  modes, whose final states are non- $CP$  eigenstates. We have calculated the  $P$ -wave contribution to the indirect

$CP$  asymmetry in  $B_d \rightarrow D^{*+}D^{*-}$  decays. The corresponding dilution effect is found to be insignificant, therefore observation of large  $CP$  violation remains under expectation even without the delicate angular analysis.

It is certainly necessary to test the validity of our factorization hypothesis, on which most of the afore-mentioned results depend. To do so a measurement of  $B_d \rightarrow D^{(*)0}\bar{D}^{(*)0}$  transitions will be particularly helpful. On the one hand, if the branching ratios of these decay modes are too small compared with those of  $B_d \rightarrow D^{(*)+}D^{(*)-}$  transitions, then the final-state rescattering effects should be negligible and the naive factorization approximation might work well. On the other hand, if the branching ratios of  $B_d$  decays into  $D^{(*)0}\bar{D}^{(*)0}$  and  $D^{(*)+}D^{(*)-}$  states are found to be more or less comparable in magnitude, then a quantitative isospin analysis should become available, allowing us to extract the isospin phase differences and control the final-state rescattering effects. In any case much can be learnt about the factorization hypothesis and its applicability in  $B$  decays into two heavy charmed mesons.

In conclusion, the observation of direct and indirect  $CP$  asymmetries in  $B_d \rightarrow D^{(*)+}D^{(*)-}$  decays is promising at  $B$ -meson factories. They are expected to provide us some valuable information about the weak phase  $\beta$  as well as the penguin and rescattering effects in non-leptonic  $B$  decays.

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